

Tesla S P100D Ludicrous Plus Mode Simulation

The Simulation can be run or modified with Mathcad 14/15. Free Trial at: <http://www.ptc.com/product/mathcad/free-trial>

Mathcad Simulation at: <http://www.LeanCad.com/TeslaSP85DLudicrousPlusModeSimulation.xmcd>

2-12-2017

Goal: Simulate Ludicrous Plus Mode Acceleration Performance

Tesla S Ludicrous Plus - Fastest Production Car in the World

This paper shows a macro model for performance simulation of a Tesla S P100D Ludicrous Plus Mode. Results were calculated for 100% and 50% State of Charge (SOC). The key parameters are peak motor torque, peak battery power (SOC), curb weight, maximum tire traction, and some assumptions about the power loss/efficiency high efficiency induction motors (93%), and Inverter and power train (87%). Net System Efficiency, SysEff ~ 81% For 50% SOC, battery max power is 780 hp, System Power of 780 hp. From statements of Elon Musk, we would infer that tire traction is capable of gripping the road @1.4g. Assume that with sufficient power, we can get a peak Motor Torque of 864 ft lb*. Under these assumptions, the model shows that we meet the 0 to 60 mph in 2.2 sec Spec for 50% SOC. This Analysis is done in the following ten Sections. Section IV considers four different traction scenarios (Sec IV, pg. 4). The Calculations & graphs in Sections III to VI @ 100% SOC. *For 864 ft lb See: http://www.greencarreports.com/news/1098611_too-much-electric-car-torque-tesla-model-s-p85d-breaks-dynamometer-video

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I. Introduction - Simple Analysis

Examining the Difference Between P100D Insane & Ludicrous Plus Speed Upgrade

Insane Mode Specifications

3.1 seconds 0-60 mph

Peak Acceleration: 1 g

Front and Back Motor Spec: Power 691 hp

Conventional Fuse: 1300 amp battery limitation

Battery Peak Power: 1300 A x 398.6 = 695 hp

Inverter and/or Algorithm sets max accel ~ 1g.

Ludicrous Mode Plus Specifications

2.3 seconds 0-60 mph with Ludicrous Speed Upgrade

Front and Back Motor Power Spec: 780 hp

Electronic Fuse (P85D): 1500 amp "effective" battery upgrade

Battery 8256 cells @50% SOC Peak Power: 1762 A x 314V = 741 hp

Peak Motor Torque: 864 ft lb.

New Inverter Algorithm to maximize Ludicrous acceleration.

We assume that the implementation of the New Algorithm for Rear Motor Torque & Power Split in Ludicrous Mode, will provide the maximum tire grip acceleration, 1.4g, and not optimize Efficiency, that is, minimize Power: $= TS_{\text{rear}} = 503/809 = 0.63$

What is the minimum acceleration needed to meet the 0 to 60 mph in 2.3 seconds? There are two factors involved in this specification, velocity, v, and time, t. For the sake of this Simple Analysis, let's assume that the acceleration is constant. Now acceleration defined as the rate of change of velocity. In symbols, the constant acceleration, a_{constant} = change of velocity/time. Then the acceleration needed to get to 60 mph in 2.3 seconds, or 60 mph/2.3 s, is a constant 26 mph per sec.

Thus a constant 26 mph/s is sufficient to meet our spec. In units of the earth's gravitation g, 1 g = 21.9 mph/s. We need a constant or "average" of 1.2 g. From Newton's 2nd Law we have that $F = m a$. Then to accelerate a mass of 4891 lb, we need a Force of 4984 lb or 14" tire radius and 9.73 Gear Ratio, a Motor Torque of 733 ft lbf. Then 864 ft lbf should be more than sufficient.

However, torque or acceleration is not constant. The 800 ft lb is peak torque. Torque falls off with vehicle speed or if the battery power is not sufficient to supply enough power to keep it at its peak. Also, Ludicrous Plus Mode uses a New Algorithm that provides complex Traction Control for max acceleration. Thus a more in-depth analysis is required. Among other things, it requires working out what the torque falloff is with speed. We will find that with a net combined Inverter and Motor Power Efficiency of 81% and high performance 1.4 g tires (implied by Elon Musk), we can meet the Ludicrous Specs. This more in-depth analysis is what follows.

II. Macro Performance Model Discussion & Description of the Model

Macro Model: Macro Models requires only limited knowledge of internal parameters. We treat the system as a Black Box. That is, we don't know the details of what's inside, just a few fundamental parameters. We are only interested in overall performance. Ignore the intricacies. Simple, but not too simple. May not know what is inside, but regardless, the laws of Physics still apply. We just need basic physical parameters such as:

Vehicle mass (M_{curb}), Coefficient of tire friction μ , and radius, Gear Ratio GR, max motor Torque & Power, battery power, and System Power Efficiency (Inverter, Gears, and Motors).

The vehicle also has rotational energy from rotating tires, motor rotor, and gear box.

A factor k_m , which multiplies the mass, accounts for this added rotational mass.

$M_{\text{curb}} = 4936 \text{ lbm}$, $\mu = 1.4$ (equivalently, max $g = 1.4$), 265/35ZR21 tires, tire radius=14 inches, GR = 9.73.

Then acceleration (a) is given by:

$$\text{Newton's Second Law: } a = \text{Traction Force}/m = \text{Torque} \times \text{GR}/M_{\text{curb}}$$

See pg 3 for Section on **Traction Control**.

Then the Torque required to get to $g = 1.4$, requires that Torque be at least:

$$\text{Torque}_{\text{max}_g} = \text{Weight} \times k_m \times 1.4 \times \text{tire radius}/\text{GR} = 857 \text{ ft lbf}$$

The present max Torque spec is 864 ft lbf. This is more than sufficient to give 1.4 g.

$$r_{\text{tire}} := 14 \text{ in}$$

$$\text{GR} := 9.73$$

$$M_{\text{curb}} := 4891 \text{ lbm}$$

$$k_m := 1.0447$$

$$T_{\text{max}_g} := M_{\text{curb}} \cdot k_m \cdot \frac{1.4g \cdot r_{\text{tire}}}{\text{GR}}$$

$$T_{\text{max}_g} = 857.73 \cdot \text{ft} \cdot \text{lbf}$$

$$T_{\text{max}} := 864 \text{ ft} \cdot \text{lbf}$$

$$\text{Power}_{\text{max}} := 780 \text{ hp}$$

$$\text{RPM}_{\text{motor}} := \frac{\text{Power}}{\text{Torque} \cdot 2 \cdot \pi}$$

Velocity at Max Power

$$v_{\text{Pmax}} := \frac{\text{Power}_{\text{max}} \cdot r_{\text{tire}}}{T_{\text{max}} \cdot \text{GR}}$$

$$v_{\text{Pmax}} = 40.59 \cdot \text{mph}$$

$$t_{\text{Pmax}} := 1.46 \text{ s}$$

What is the Estimated Motor Power needed to meet the 0 - 60 mph in 2.8 s performance, P_{Spec} ?

Ludicrous Mode: The current has been increased from 1300 to 1500 amps (15 %), the total dual motor power increased from 682 hp to **762 hp** (12%), and the total torque from 707 to 713 ft lbf. However, as we saw in section I, the System Power to the motor @81% System Efficiency is limited to **649 hp**

There is a basic relationship between Torque, Motor RPM, and Motor Power:

Assume that there is No Traction Control, this is tires can slip.

Thus initially, full torque is applied to the wheels, until max motor power limits the torque.

Refer to the below plot and examine the Power versus time profile.

The Power is given by: Power = Torque x Angular Velocity, until Max Power is reached.

This is shown in the graph below. Tire velocity, v_{Pmax} , to get to max motor power and Torque =

The time to get to max motor power is t_{Pmax} . The velocity at which this occurs is v_{Pmax} .

There are 2 paths to get to the max power: #1 Tires allowed to slip and #2 Tires do not slip (Pg. 3).

Elon Musk said that the peak acceleration is 1.4 g. Designate the time to reach full power, t_{Pmax} .

$$P_m(\omega) = T(\omega) \cdot k \cdot 2 \cdot \pi \cdot \omega \text{ RPM}$$

The average power from the start at 0 power to the peak of $\text{Power}_{\text{max}}$ is $1/2 \text{ Power}_{\text{max}}$.

If the time to get motor Power max is t_{Pmt} , then the Energy is $1/2 \text{ Power}_{\text{max}} \times t_{\text{Pmax}}$.

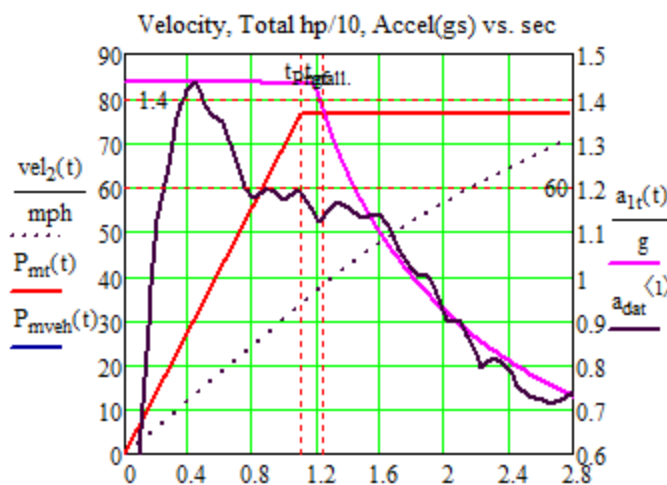
This energy is equal to the Kinetic Energy in going from zero velocity to velocity, v.

The relationship; is shown at the right.

The most critical part

of performance is the 1.2 seconds of Peak Power from the time the acceleration falls from 1.4 g to the 2.3 second spec limit. The black curve is Motor Trend's acceleration (gs) test data.

Note: hp data is divided by 10 to use the same scale as the velocity in mph.



The 2 motor power curves belong to two different Traction Models:

#1: No Traction Control, Red Curve

#2: With Traction Control, Blue Curve

t_{Pmt} is the time to Peak Power = 1.46s

t_{gfall} is the time for acceleration(g)

to fall below 1.4g = 1.2 s.

These times are in seconds.

$$\frac{1.46 + 1.69}{2} = 1.58$$

$$M_{\text{gross}} := M_{\text{curb}} + 160 \text{ lbm}$$

$$\text{KE}(v) := \frac{1}{2} \cdot M_{\text{gross}} \cdot k_m \cdot (v)^2$$

$$\text{KE}(60 \text{ mph}) = 0.24 \cdot \text{kW} \cdot \text{hr}$$

What Max Power do we need to meet the 0 - 60 mph in 2.3 s with No Traction Control?

For a vehicle velocity, v, the the Kinetic Energy, KE, to get to v is given by:

$$E_{\text{motor}} := \frac{1}{2} \cdot 780 \text{ hp} \cdot 1.2 \text{ s} + 649 \text{ hp} \cdot (2.3 - 1.2) \text{ sec} = 0.24 \cdot \text{kW} \cdot \text{hr}$$

This demonstrates that 780 hp is sufficient to meet the 0 - 60 mph in 2.3 sec spec.

What follows is a more detailed analysis.

III. Specifications & Engineering Estimates: Ludicrous Mode Peak Acceleration

System Efficiency: SysEff := 0.90 % SOC Voltages: $V_{batt_100} := 394\text{volt}$ $V_{batt_80} := 384\text{volt}$ $V_{batt_50} := 360\text{volt}$ $V_{batt} := V_{batt_50}$

Results are Shown for **100% State of Charge.**

Battery and System Power **@100% SOC**

$$\text{Power}_{\text{Batt}} := V_{\text{batt}} \cdot 1762\text{A} = 850.64 \cdot \text{hp}$$

$$\text{Power}_{\text{System}} := \text{Power}_{\text{Batt}} \cdot \text{SysEff} = 765.57 \cdot \text{hp}$$

85 kW-hr Battery

$$\text{De-acceleration Battery Energy Regeneration Factor: Regen} := 0.64$$

New Algorithm for Rear to Front Motor Torque & Power Split to maintain **peak acceleration**: $T_{\text{Split}} = 503/259 \sim 1.94$ Gear Ratio:

Ludicrous + Mode Max Power: $\text{Power}_{\text{max}} := 741 \cdot \text{hp}$ $\text{RPM}_{\text{max}} := 18000$ $T_{\text{Split}} := 1.94$ $\text{GR} := 9.73$

Limit: $\text{Power}_{\text{System}} = 765.57 \cdot \text{hp}$

$$\text{Power}_{\text{max}} := \text{Power}_{\text{System}}$$

Battery Energy:

$$\text{Energy}_{\text{bat}} := 100 \cdot \text{kW} \cdot \text{hr} \quad R_{\text{phase}} := 0.006 \text{ohm}$$

Max Dual Motor Torque:

$$\text{Power}_{\text{max}} = 765.57 \cdot \text{hp}$$

From TeslaMotors.com/models

$$T_{\text{max}} := 864 \cdot \text{ft} \cdot \text{lb} \cdot \text{ft}$$

Torque_{maxOld} := 707 · ft · lbf

245/35R19 Tire Radius:

$$r_{\text{tire}} := \frac{26.3}{2} \text{in}$$

Ultra High Performance Tire

$$F_{\text{Motor_Max}} := \frac{T_{\text{max}} \cdot \text{GR}}{r_{\text{tire}}}$$

Tire Coefficient of Friction, μ :

$$\mu := 1.4$$

Max 1.4 g Quoted by Elon Musk

$$\text{car}_{\text{max_g}} := \mu \cdot g$$

$$k := 1000$$

$$\tau := 1 \cdot \text{sec}$$

$$\text{Curb Weight: } M_{\text{curb}} := 4891 \text{lbm} \quad M_{\text{gross}} := M_{\text{curb}} + 160 \text{lbm} = 5051 \cdot \text{lbm}$$

Aerodynamic Drag Coeff (TM):

$$C_d := 0.22$$

Average Wind Velocity:

$$V_w := 0 \cdot \text{mph}$$

$$g_{\text{max}} := \frac{T_{\text{max}} \cdot \text{GR}}{M_{\text{gross}} \cdot k_m \cdot r_{\text{tire}} \cdot g}$$

Cross Wind Drag Coff:

$$C_{d_{\text{cw}}} := 0.000014$$

Effective Cross Wind V:

$$V_{\text{cw}} := 0 \cdot \text{mph}$$

Shape Correction Factor:

$$\text{SCF} := 0.85$$

Vehicle Frontal Dimensions:

$$A_f := (57 - 7.9) \text{in} \cdot 77 \cdot \text{in} \quad g_{\text{max}} = 1.45$$

Air Density, tire resistance:

$$\rho := 1.293 \cdot \frac{\text{gm}}{\text{liter}}$$

Drag Frontal Area

$$A_d := A_f \cdot \text{SCF}$$

$$A_d = 2.07 \cdot \text{m}^2$$

Road Rolling Resistance:

$$\text{RR}_{\text{road}} := 0.007$$

Tire Rolling Resist, Hys:

$$\text{RR}_{\text{tire}} := 0.011$$

$$T_{\text{hys}} := 0 \cdot \frac{\text{sec}}{\text{m}}$$

Effective Mass Coefficient:

$$k_m := 1.0447$$

EPA Range Spec for P100 is 315 miles See page 6 for

$$\text{EPA_RangeSim} := 251 \text{mile}$$

IV. Tire Traction & Control Models: #1 Perfect Grip, #2 Tires slip, #3 No Slip, #4

Simple Step Model of Tire Traction (Assume perfect weight distribution per motor, i.e. same acceleration at each motor)

Depending on road conditions, Tires do not have perfect grip, they may slip. Vehicle acceleration, a_{veh} is limited to the maximum tire traction ($\text{tire}_{\text{max_g}} = 1.4g$). The tire rpm x GR = motor rpm, but because of slip, tire velocity can be greater than vehicle velocity. Therefore, vehicle acceleration and velocity are not directly proportional to rpm, that is, tires may slip:

Case #1

$$v_{\text{tire}} = v_{\text{vehicle}}$$

No Traction Control, but no tire slip.

Max motor power and torque are applied to tires. Perfect Tires that do not slip.

Acceleration can exceed 1.4 g.

Case #2

$$v_{\text{tire}} = v_{\text{vehicle}}$$

Perfect Traction System and High Performance $g = 1.4$ tires.

Because of Traction Control and tire slip, effective motor rpm can be greater than vehicle speed during tire slip or Traction Control. Vehicle speed depends on tire coefficient of resistance, μ , which is equal to 1.4 for Ludicrous+. This allows a **max of 1.4 g**. For Case #2, we assume Traction Control limits g to 1.4.

Macro Model of Motor Dynamics: Velocity of Tire is v

Angular Velocity Symbol, Ω (units of radians/second)	$\Omega(\omega) := 2\pi 1000 \cdot \omega \cdot \text{min}^{-1}$	RPM/1000 Symbol, ω_k	$\text{RPM} := \text{min}^{-1}$
Angular Vel Ω @Max Power:	$\Omega_{\text{Pmax}} := \text{Power}_{\text{max}} \cdot T_{\text{max}}^{-1}$	$\text{RPM}_{\text{Pmax}} := \frac{\Omega_{\text{Pmax}}}{2 \cdot \pi}$	$\text{RPM}_{\text{Pmax}} = 4653.79 \cdot \text{RPM}$
Convert velocity to RPM	$V_{\text{toRPM}}(v_v) := v_v \cdot (1000 \cdot 2 \cdot \pi \cdot r_{\text{tire}} \cdot \text{RPM})^{-1}$	$\omega_{\text{Pfall}} := \text{RPM}_{\text{Pmax}} \cdot k^{-1} = 4.65 \cdot \text{RPM}$	
Tire Velocity at Torque Fall:	$v_{\text{Tfall}} := \text{RPM}_{\text{Pmax}} \cdot 2 \cdot \pi \cdot r_{\text{tire}} \cdot \text{GR}^{-1}$	$v_{\text{Tfall}} = 37.42 \cdot \text{mph}$	
Tire Velocity to kRPM:	$V_{\text{toKR}}(v_t) := v_t \cdot (k \cdot 2 \cdot \pi \cdot r_{\text{tire}} \cdot \text{RPM})^{-1}$	$V_{\text{toKR}}(60 \cdot \text{mph}) \cdot \text{GR} = 7.46$	
Road Resistance, Ft:	$F_t(v_v) := M_{\text{gross}} \cdot g \cdot [T_{\text{hys}} \cdot v_v \cdot \sin(\theta) + (\text{RR}_{\text{tire}} + \text{RR}_{\text{road}}) \cdot \cos(\theta) + \sin(\theta)]$	RPM_{Pmax} for Max Power:	
Air Drag Force, Fa:	$F_a(v_v) := 0.5 \cdot \rho \cdot A_d \cdot [(v_v + V_w)^2 \cdot C_d + C_{d_{\text{cw}}} \cdot (V_{\text{cw}})^2]$	Note: For Drag and Road Resistance, approximate vehicle with v_{tire} . At	
Total Opposing Force, Fo:	$F_o(v_v) := F_a(v_v) + F_t(v_v)$	$F_o(60 \cdot \text{mph}) = 138.61 \cdot \text{lbf}$	<60mph Compared to Ftire, Fo is small.
Torque/Force Falloff Curve:	$\omega_{\text{kmax}} := 15.8 \cdot \text{RPM}$	$T_{\text{PLt}}(\omega_k) := \text{Power}_{\text{max}} \cdot \Omega(\omega_k)^{-1}$	$T_{\text{PLt}}(55) = 73.11 \cdot \text{ft} \cdot \text{lbf}$
Tm is Torque of motor	$T_m(\omega_k) := \text{if}(\omega_k \cdot \text{RPM} \geq \omega_{\text{Pfall}}, T_{\text{PLt}}(\omega_k), T_{\text{max}})$	$P_m(\omega_k) := T_m(\omega_k) \cdot k \cdot 2 \cdot \pi \cdot \omega_k \cdot \text{RPM}$	
Fmot, Tractive Force from motor, not from slipping tires:	$T_{\text{mv}}(v_t) := T_m(V_{\text{toKR}}(v_t) \cdot \text{GR})$	$F_{\text{mot}}(v_t) := \frac{\text{GR}}{r_{\text{tire}}} \cdot T_{\text{mv}}(v_t)$	$F_{\text{PL}}(v_t) := \text{Power}_{\text{max}} \cdot (v_t \cdot \text{mph})^{-1}$

Solve for Velocity, Acceleration, and Distance versus Time

We are using Mathcad 14, a Computer Math Program, to do the Calculations: <http://www.ptc.com/product/mathcad/free-trial>

Case 1: Perfect Grip Tires at Maximum Motor Power, Coefficient of Tire Friction > 1.4 g

Newton's Third Law of Motion:

$$a_1(v) := \frac{F_{\text{mot}}(v) - F_o(v)}{k_m \cdot M_{\text{gross}}} \quad a_{1T\text{max}} := \frac{T_{\text{max}} \cdot \text{GR}}{M_{\text{gross}} \cdot k_m \cdot r_{\text{tire}}} = 1.45 \cdot g$$

$$V := 0 \cdot \text{mph} \quad \text{vel}_1(t) := \text{root} \left(t \cdot \text{sec} - \int_0^V \frac{\text{mph}}{a_1(V \cdot \text{mph})} dV, V \right) \cdot \text{mph} \quad \text{time}_{a1}(v) := \int_0^v \frac{1}{a_1(v)} dv$$

$$\text{time}_{a1}(60\text{mph}) = 2.13 \text{ s} \quad \text{vel}_1(2.22) = 61.71 \cdot \text{mph}$$

$$v_{\text{gfall}} := \text{root}(a_1(V \cdot \text{mph}) - \text{car}_{\text{max}_g}, V) = 38.29 \quad \text{time}_{a1}(60\text{mph}) = 2.13 \text{ s} \quad a_{1t}(t) := a_1(\text{vel}_1(t))$$

$$\text{Velocity } g \text{ fall, } a \leq 1.4g \quad a_1(v_{\text{gfall}} \text{ mph}) = 1.4 \cdot g \quad a_{1t}(0) = 1.44 \cdot g$$

Case 2: High Performance 1.4 g Tires and Acceleration Limited to 1.4 g, but Use Max Power and No Spin

a_2 acceleration is allowed by high performance tires on dry road.

$$a_2(v) := \text{if}(a_1(v) \geq \text{car}_{\text{max}_g}, \text{car}_{\text{max}_g}, a_1(v)) \quad \text{car}_{\text{max}_g} = 1.4 \cdot g$$

$$\text{vel}_2(t) := \text{root} \left(t \cdot \text{sec} - \int_0^V \frac{\text{mph}}{a_2(V \cdot \text{mph})} dV, V \right) \cdot \text{mph} \quad \text{time}_{a2}(v) := \int_0^v \frac{1}{a_2(v)} dv$$

$$\text{vel}_2(2.26) = 61.89 \cdot \text{mph}$$

a as function of time, not velocity $a_{2t}(t) := a_2(\text{vel}_2(t)) \quad t_{\text{gfall}} := \text{time}_{a2}(v_{\text{gfall}} \cdot \text{mph}) = 1.25 \text{ s} \quad a_{2t}(0\text{mph}) = 1.4 \cdot g$

$$\text{time}_{a2}(60\text{mph}) = 2.16 \text{ s}$$

$\text{distance}_2(10.) = \text{■} \cdot \text{mile} \quad \text{distance}_2(10.46) = \text{■} \cdot \text{mile}$

RPM at g fall: $R_{\text{gfall}} := V_{\text{tokR}}(v_{\text{gfall}} \cdot \text{mph}) \cdot \text{GR}$

Case 3: Traction Control - Tire Force is Power Limited - No Tire Spin (This model not yet perfected)

$$F_{1_1g} := k_m \cdot M_{\text{gross}} \cdot \text{car}_{\text{max}_g} = 7387.49 \cdot \text{lbf} \quad T_{1_1g} := \frac{F_{1_1g} \cdot r_{\text{tire}}}{\text{GR}}$$

$$T_3(\omega_k) := \text{if}(\omega_k \leq R_{\text{gfall}}, T_{1_1g}, \text{Power}_{\text{max}} \cdot \Omega(\omega_k)^{-1})$$

$$P_{1_1g}(\omega_k) := T_{1_1g} \cdot \omega_k \cdot k \cdot 2 \cdot \pi \cdot \text{RPM} \quad P_{1_1g}(5.252) = 831.99 \cdot \text{hp} \quad \omega_{3p\text{max}} := 5252$$

$$P_3(\omega_k) := T_3(\omega_k) \cdot (\omega_k \cdot k \cdot 2 \cdot \pi \cdot \text{RPM}) \quad F_3(v) := \frac{\text{GR}}{r_{\text{tire}}} \cdot T_3(V_{\text{tokR}}(v \cdot \text{mph}) \cdot \text{GR})$$

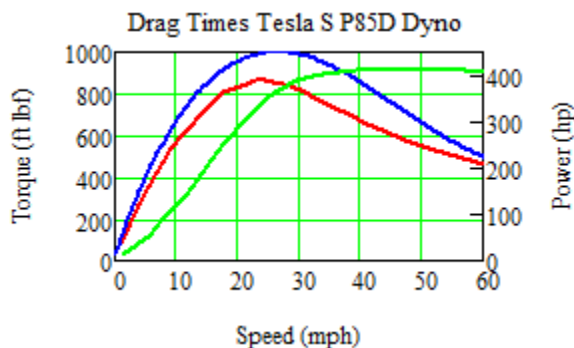
Case 3: We end up getting the same effective peak torque, we just don't waste the power put into spinning wheels. Tesla has a patent on how to split the power

Case 4: Fit Torque to Drag-Times Dyno Torque Curve Shape -->Even @100% Efficiency, Does Not Meet Specs

$a_4(v) = \text{P85D Acceleration/Torque, } a_1(v) \times T_{\text{Shape}}(v):$

$$a_4(v) := \text{if} \left(a_1(v) \cdot T_{\text{Shape}} \left(\frac{v}{\text{mph}} \right) \geq \text{car}_{\text{max}_g}, \text{car}_{\text{max}_g}, a_1(v) \cdot T_{\text{Shape}} \left(\frac{v}{\text{mph}} \right) \right)$$

$T_{\text{Shape}}(v)$ was fitted to have the same shape, as the Drag Times Torque/speed Curve - See Graph Below



Dyno Data by Drag Times for 2015 Tesla S P85D

Dyno Torque (Red) and Dyno Power (Green)

DynoTorque Shape x 1000 (Blue), Extracted from Dyno Torque

<http://www.dragtimes.com/2015-Tesla-Model-S-Videos-27143.html>

$$\text{vel}_4(t) := \text{root} \left(t \cdot \text{sec} - \int_0^V \frac{\text{mph}}{a_4(V \cdot \text{mph})} dV, V \right) \cdot \text{mph} \quad \text{vel}_4(t) := \text{vel}_4(t) \cdot \text{mph}^{-1}$$

$$a_{4t}(t) := a_4(\text{vel}_2(t)) \quad \text{time}_{a4}(v) := \int_0^v \frac{1}{a_4(v)} dv$$

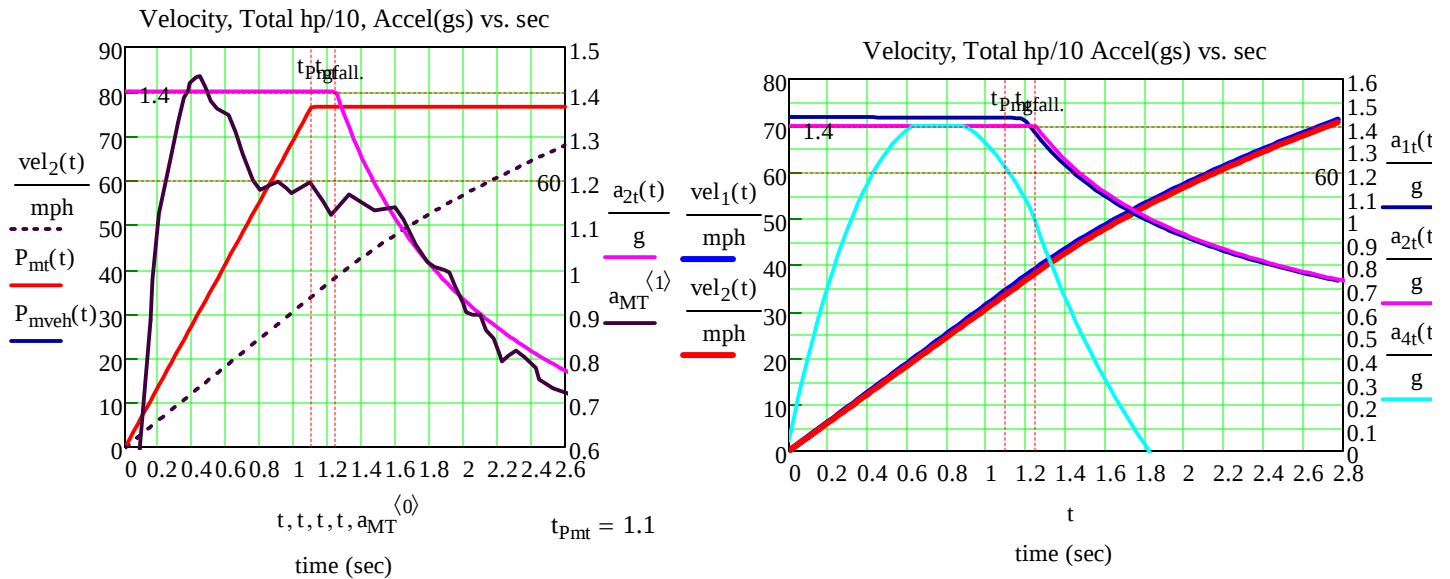
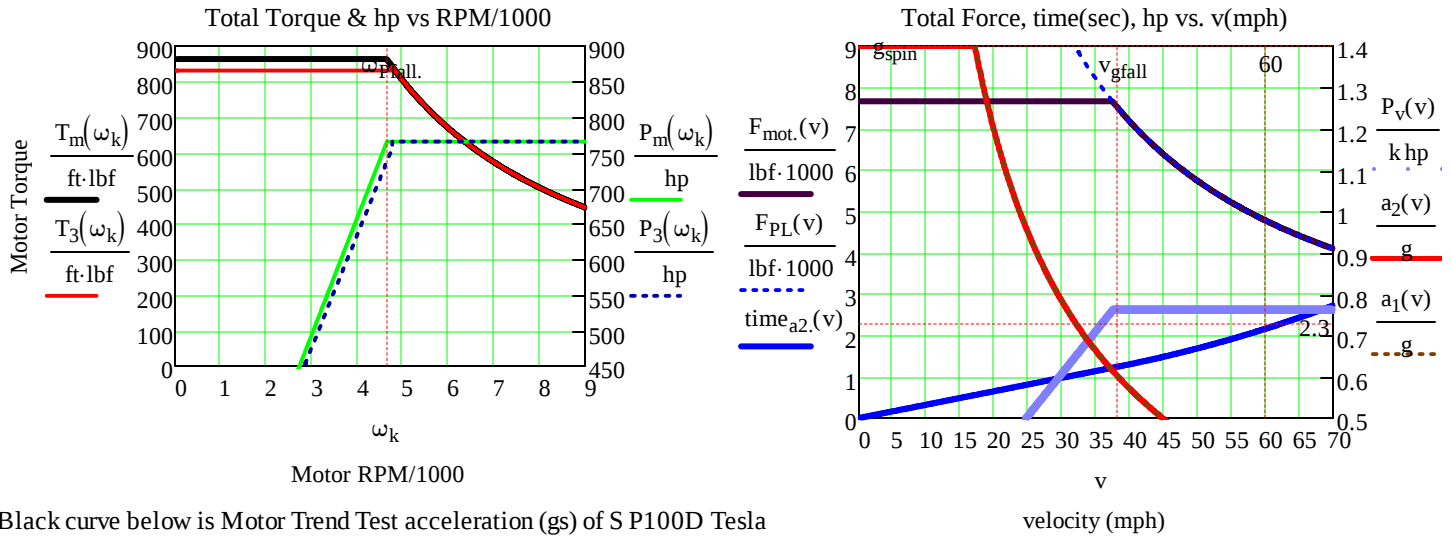
Does not meet Specs
 $\text{time}_{a4}(60\text{mph}) = \text{■}$

V. Model Results and Validation Meets Ludicrous Specs

$\text{time}_{a2}(60 \cdot \text{mph}) = 2.16 \text{ s}$

$\text{distance}_2(10) = \text{■} \cdot \text{mile}$

Calculated P100 EPA Range: 297 Miles



VII. Find the Single Charge. Highway Cruise Range for a Given Velocity and Final SOC

Driving Pattern/Profile: Assume we cruise at constant speed, but start, stop, and regen break four times per hour

Drive Train Power Efficiency - Battery Loss for Commanded Vehicle Velocity and Final State of Charge, SOC_f :

SOC_f is 10% at recharge. 400V HV battery idle power is P_o . 12V battery gives Accessory Power. The Traction Inverter Efficiency - TI_{InvE} , HV Power Electronics at Idle Efficiency - $IPEE$, and Gear Power Efficiency - GPE are 92.5%, 95%, and 90%, respectively. Brake Regen efficiency of kinetic energy is 64%. Then the number of starts per hour as a function of velocity, NS , $NumStarts(v, P_o)$, is

$$TI_{InvE} := 0.925 \quad IPEE := 0.95 \quad GPE := 0.9 \quad \text{Regen} := 0.64 \quad \text{Change in State of Charge} = 1 - SOC_f$$

$$Power_{dissLoss}(v, P_o) := \frac{Fo(v) \cdot v}{TI_{InvE} \cdot GPE} + \frac{P_o \cdot \text{watt}}{IPEE} \quad Energy_{accel}(v) := Power_{max} \cdot \text{time}(v, \text{mph}) \cdot \text{hr}$$

NS_o , NS are iterative converging

estimates of total $NumStarts$ per charge

$$NS_o(v) := 2 \cdot \left[\frac{65 \text{mph}}{(v + 0.1 \cdot \text{mph})} \right]^2 \quad NS(v, P_o, SOC_f) := \frac{Energy_{bat} \cdot (1 - SOC_f) - NS_o(v) \cdot \left[\frac{M_{gross} \cdot (v)^2}{2} (1 - \text{Regen}) \right]}{Power_{dissLoss}(v, P_o) \cdot 15 \cdot \text{min}}$$

$$NumStarts(v, P_o, SOC_f) := \text{floor} \left[\frac{Energy_{bat} \cdot (1 - SOC_f) - NS(v, P_o, SOC_f) \cdot \left[\frac{M_{gross} \cdot (v)^2}{2} (1 - \text{Regen}) \right]}{Power_{dissLoss}(v, P_o) \cdot 15 \cdot \text{min}} \right]$$

$$Cruise_Range(v, P_o, SOC_f) := \left[\frac{Energy_{bat} \cdot (1 - SOC_f) - NumStarts(v, P_o, SOC_f) \cdot \left[\frac{\text{Regen} \cdot M_{gross} \cdot (v)^2}{2} (1 - \text{Regen}) \right]}{Power_{dissLoss}(v, P_o)} \right] \cdot v$$

Highway Cruise Range with Four Stops per Hour Estimate

Cruise_Range(30·mph, 100, 0.1) = 358.73·mi

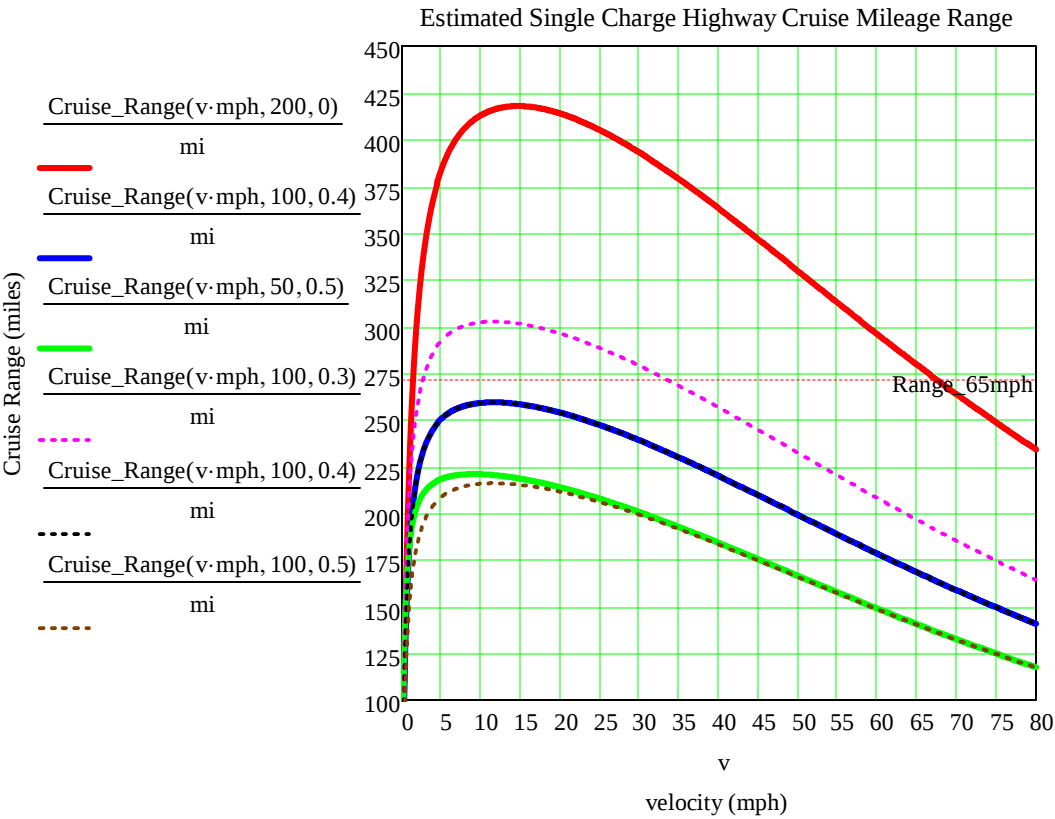
Cruise_Range(40·mph, 100, 0.1) = 330.03·mi

Cruise_Range(50·mph, 100, 0.1) = 298.8·mi

Cruise_Range(60·mph, 100, 0.1) = 267.7·mi

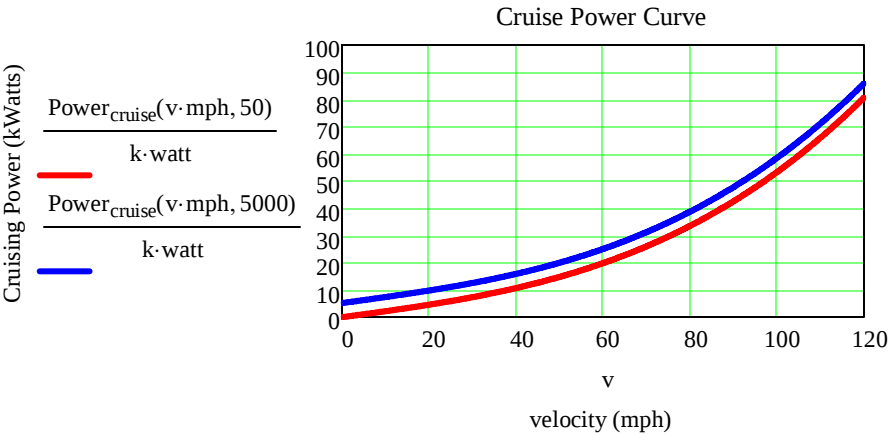
Cruise_Range(70·mph, 100, 0.1) = 238.32·mi

Cruise_Range(60·mph, 200, 0) = 295.87·mi



Opposing Force Power Dissipation

$$\text{Power}_{\text{cruise}}(v, P_o) := \text{Power}_{\text{dissLoss}}(v, P_o)$$
$$\text{Power}_{\text{cruise}}(60 \cdot \text{mph}, 500) = 2.04 \times 10^4 \cdot \text{watt}$$



VIII. Find Mileage Range: Use 3 Different EPA Driving Schedules

Algorithm to Calculate Range, Range(P,fHz), 100% Battery Discharge, Driving Profile Velocity/Time File, P and Sampling Rate, fHz

$$\text{Energy}_{\text{bat}} := 100 \cdot \text{kW} \cdot \text{hr}$$

$$\begin{aligned} \text{Range}(P, f_{\text{Hz}}) := & \begin{array}{|l} \text{Ebat} \leftarrow \text{E}_{\text{diss}} \leftarrow v_{\text{old}} \leftarrow 0 \\ n \leftarrow -1 \\ N \leftarrow \text{rows}(P) - 1 \\ \text{while} \left(\text{E}_{\text{diss}} < \frac{\text{Energy}_{\text{bat}}}{\text{kW} \cdot \text{hr}} \right) \\ \quad n \leftarrow n + 1 \\ \quad t \leftarrow \text{mod}(n, N) \\ \quad v \leftarrow P_t \\ \quad P_{\text{accel}} \leftarrow \frac{k_m \cdot M_{\text{gross}} \cdot (v^2 - v_{\text{old}}^2) \cdot \frac{\text{mph} \cdot f_{\text{Hz}}}{\text{sec}} \cdot \text{mph}}{T_{\text{InvE}} \cdot \text{GPE} \cdot 2} \quad \text{if } v > v_{\text{old}} \\ \quad P_{\text{accel}} \leftarrow k_m \cdot M_{\text{gross}} \cdot (v^2 - v_{\text{old}}^2) \cdot \frac{\text{mph} \cdot f_{\text{Hz}}}{2 \text{sec}} \cdot \text{mph} \cdot \text{Regen} \quad \text{otherwise} \\ \quad \text{E}_{\text{diss}} \leftarrow \text{E}_{\text{diss}} + \frac{(\text{Power}_{\text{dissLoss}}(v \cdot \text{mph}, 100) + P_{\text{accel}}) \cdot \text{sec}}{\text{kW} \cdot \text{hr} \cdot f_{\text{Hz}}} \quad \begin{array}{l} \text{If decelerating, charge battery with} \\ \text{Regen fraction of energy.} \end{array} \\ \quad v_{\text{old}} \leftarrow v \\ \quad \text{Ebat}_n \leftarrow \text{E}_{\text{diss}} \\ \text{Range} \leftarrow \sum_{m=0}^n \frac{(P_{\text{mod}(m, N)} + P_{\text{mod}(m+1, N)}) \cdot \text{mph} \cdot \text{sec}}{2 \cdot \text{mi} \cdot f_{\text{Hz}}} \end{array} \end{aligned}$$

Read US06 and FTP Dynamometer Drive Profile Files

Refer to: <http://www.epa.gov/nvfel/testing/dynamometer.htm>

The US06 cycle represents an 8.01 mile (12.8 km) route with an average speed of 48.4 miles/h (77.9 km/h), maximum speed 80.3 miles/h (129.2 km/h), and a duration of 596 seconds. Sampling can be either 1 Hz or 10Hz

The **Federal Test Procedure (FTP)** is composed of the UDDS followed by the first 505 seconds of the UDDS. It is often called the EPA75. 10 Hz Sampling data is named FP10 and HY 10 for the Highway schedule.

$$\begin{aligned} \text{FTPF} &:= \text{READPRN}(\text{"FedTestProc.txt"}) & t &:= \text{FTPF}^{\langle 0 \rangle} & \text{FTP} &:= \text{FTPF}^{\langle 1 \rangle} & \text{rows}(\text{FTP}) &= 1875 \\ \text{UDDSF} &:= \text{READPRN}(\text{"uddscol.txt"}) & & & \text{UDDS} &:= \text{UDDSF}^{\langle 1 \rangle} & \text{rows}(\text{UDDS}) &= 1370 \\ \text{HWYF} &:= \text{READPRN}(\text{"hwycol.txt"}) & & & \text{HWY} &:= \text{HWYF}^{\langle 1 \rangle} & R_{\text{hwy}} &:= \text{rows}(\text{HWY}) \\ \text{FP10} &:= \text{READPRN}(\text{"FTP10Hz.TXT"}) & \text{FTP10V} &:= \text{submatrix}(\text{FP10}, 0, \text{rows}(\text{FP10}) - 1, 1, \text{cols}(\text{FP10}) - 1) \\ \text{HY10} &:= \text{READPRN}(\text{"HWY10Hz.TXT"}) & \text{HWY10V} &:= \text{submatrix}(\text{HY10}, 0, \text{rows}(\text{HY10}) - 1, 1, \text{cols}(\text{HY10}) - 1) \\ \text{US06F} &:= \text{READPRN}(\text{"US06PROFILE.TXT"}) & \text{Time} &:= \text{US06F}^{\langle 0 \rangle} & \text{US06} &:= \text{US06F}^{\langle 1 \rangle} & n_6 &:= 0..598 \\ r1 &:= 0.. \text{rows}(\text{HY10}) \cdot 10 - 1 & \text{HWY10}_{r1} &:= \text{HWY10V} & \text{ceil} &\left(\frac{r1+1}{10} \right) - 1, \text{mod}(r1, 10) \end{aligned}$$

Using EPA Profiles and above Range Program, Calculate Tesla EV Range for EPA Profiles

$$\text{Range}_{\text{US06}} := \text{Range}(\text{US06}, 1) \quad \text{Range}_{\text{FTP}} := \text{Range}(\text{FTP}, 1) \quad \text{Range}_{\text{HWY}} := \text{Range}(\text{HWY}, 1)$$

EPA 2008 Cycle MPG Fuel Economy Least Squares Fit Regression for Range

$$\text{MPG}_{\text{city}} := \frac{1}{\left(0.003259 + \frac{1.18053}{\text{Range}_{\text{FTP}}}\right)} \quad \text{MPG}_{\text{hwy}} := \frac{1}{0.001376 + \frac{1.3466}{\text{Range}_{\text{HWY}}}}$$

$$\text{MPG}_{\text{epa}} := 0.55 \cdot \text{MPG}_{\text{city}} + 0.45 \cdot \text{MPG}_{\text{hwy}}$$

Single Charge EPA Range Calculations: Federal Test Procedure (FTP), Highway, and US06

Model Validation:

Published EPA Range is 315 miles

$$\text{Range}_{\text{FTP}} = 272.46$$

$$\text{Range}_{\text{HWY}} = 290.79$$

$$\text{Range}_{\text{US06}} = 213.33$$

$$\text{MPG}_{\text{city}} = 131.72$$

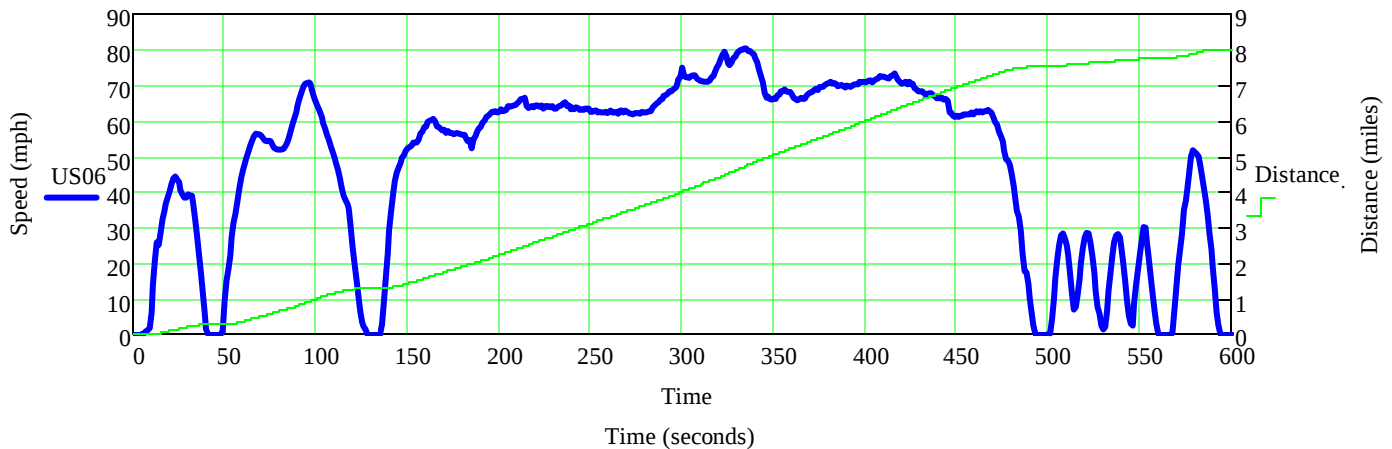
$$\text{MPG}_{\text{hwy}} = 166.48$$

$$\text{MPG}_{\text{epa}} = 147.36$$

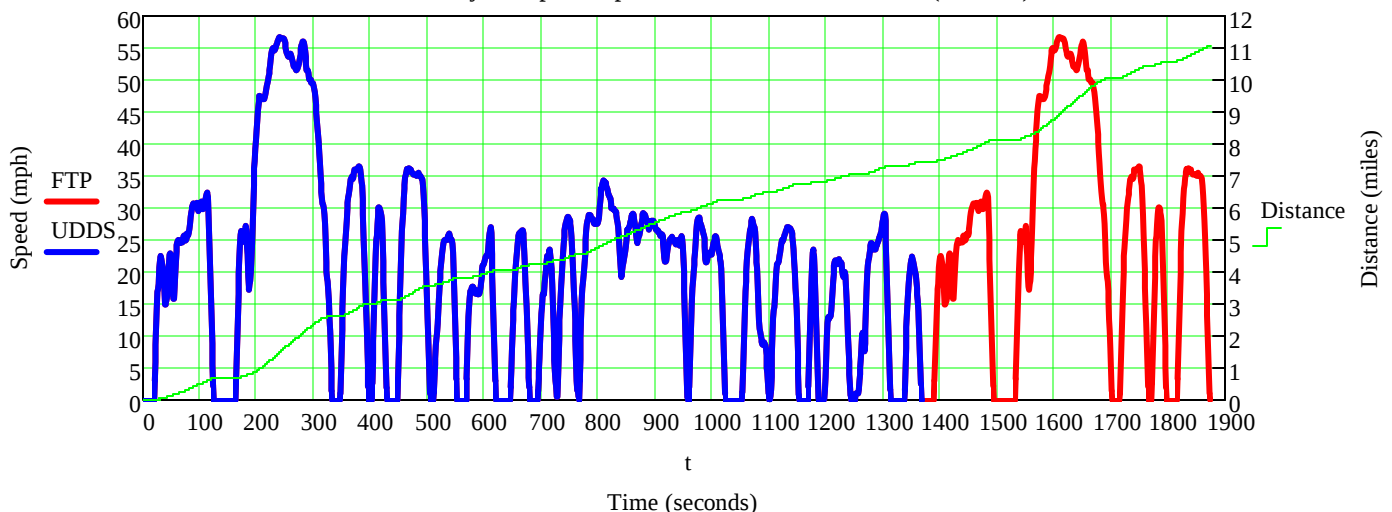
$$\begin{aligned} r &:= 0.. \text{rows}(\text{FTP}) - 1 & \text{Distance}_r &:= \sum_{r=0}^r \text{FTP}_r \cdot \frac{1}{60 \cdot 60} & rr &:= 0.. \text{rows}(\text{US06}) - 1 & \text{Distance}_{rr} &:= \sum_{rr=0}^{rr} \text{US06}_{rr} \cdot \frac{1}{60 \cdot 60} \\ \max(\text{Distance}) &= 11.04 & \max(\text{Distance}) &= 11.04 & \max(\text{Distance}) &= 8.01 \end{aligned}$$

Plots of EPA Dynamometer Vehicle Testing Profiles

US06 Drive Cycle: Speed mph and Distance miles vs time (seconds)



FTP Drive Cycle: Speed mph and Distance miles vs time (seconds)



X. Drag Times Tesla Torque and Power Dynamometer

<http://www.motortrend.com/cars/tesla/model-s/2017/2017-tesla-model-s-p100d-first-test-review/>



Extract Points from Curves to csv data files

Read Dyno Data csv data files

$T_{Dyn} := \text{READPRN}(\text{"Tesla P85D DynoTorq Insane.csv"})$ $T_{Dvn}^{(0)} := T_{Dvn}^{(0)} - 5.916$ $\text{rows}(T_{Dyn}) = 27$
 $P_{Dyn} := \text{READPRN}(\text{"Tesla P85D DynPowerR.csv"})$ $P_{Dvn}^{(0)} := P_{Dvn}^{(0)} - 5.916$ $\text{rows}(P_{Dyn}) = 24$

Power Fit Torque to get Shape

Guess: $a := 1$ $b := 1$ $c := 1$ $d := 27$ $e := 1$ $f := 30$ $gx := 0$

Given $T_{Dvn} := T_{Dvn}^{(1)}$ $spd := T_{Dvn}^{(0)}$

$\text{Torq} - [a \cdot (spd)^3 - c \cdot (spd - d)^2 - e \cdot (spd - f) + gx] = 0$ $At := \text{Minerr}(a, c, d, e, f, gx)$

$\text{Torque}(s) := At_0 \cdot (s)^3 - At_1 \cdot (s - At_2)^2 - At_3 \cdot (s - At_4) + At_5$ $\text{Torque}(0) = 23.24$

Guess $a := 1$ $b := 1$ $c := 1$ $d := 1$ $e := 1$ $f := 30$ $gx := 0$ $h := 0$ $i := 0$

Given $P_{wheel} := P_{Dyn}^{(1)}$ $spd := P_{Dyn}^{(0)}$ $n := 0..159$ $S_n := n \cdot 0.5$ $T_{x_n} := \text{Torque}(S_n)$

$P_{wheel} - [a \cdot (spd - b)^3 - c \cdot (spd - d)^2 - e \cdot (spd - f) + gx \cdot (spd - h) + i] = 0$ $A1 := \text{Minerr}(a, b, c, d, e, f, gx, h, i)$

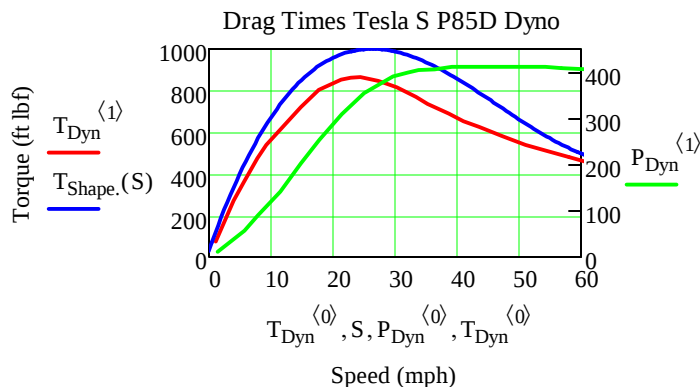
$Pwr(s) := A1_0 \cdot (s - A1_1)^0 - A1_2 \cdot (s - A1_3)^2 - A1_4 \cdot (s - A1_5) + A1_6 \cdot (s - A1_7) + A1_8$ $Pwr(30) = 344.2$

$P_{Dyn_Fv} := \left(T_{Dyn}^{(1)} \cdot \frac{\text{ft} \cdot \text{lb} \cdot \text{f}}{\text{r}_{tirex} \cdot \text{in}} \cdot T_{Dyn}^{(0)} \cdot \frac{\text{mph}}{14 \cdot \text{hp}} \right)$ $P_{Dyn_Fv_{20}} = 177.41 \text{ m}^{-1}$

Normalize Torque Curve to Max = 1 to extract shape only, TShape

Torque Shape: $\text{NormT} := \frac{1}{\max(Tx)}$ $T_{Shape}(s) := \text{Torque}(s) \cdot \text{NormT}$ $T_{Shape}(s) := T_{Shape}(s) \cdot 1000$

Multiple by 1000 to plot on same axis with Original Torque Curve



$$At = \begin{pmatrix} 0.01 \\ 1.88 \\ 7.86 \\ -41.57 \\ -154.22 \\ -6270.96 \end{pmatrix} \quad A1 = \begin{pmatrix} 0 \\ 6.65 \\ 0.36 \\ -21.66 \\ 9.2 \\ 278.18 \\ 49.07 \\ 18.22 \\ -1557.66 \end{pmatrix}$$

IX. Tire Friction (Composition and Width)

Coefficient of Static Friction (μ) is the ratio of Tire Road Force to Vehicle Weight. Values of μ for Conventional Car tire On: Asphalt 0.72, Car tire Grass 0.35.

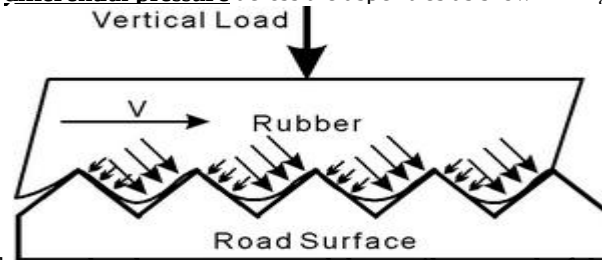
Top Fuel drag car tires are getting a coefficient of friction well over **4.5. How is this possible?**

This material came from: <http://insideracingtechnology.com/tirebkxerpt1.htm>

See Mathcad/EVs/Tire Friction.doc

Rubber generates friction in three major ways: **adhesion, deformation, and wear.**

Rubber in contact with a smooth surface (glass is often used in testing) generates friction forces mainly by adhesion. When rubber is in contact with a rough surface, another mechanism, deformation, comes into play. Movement of a rubber slider on a rough surface results in the deformation of the rubber by high points on the surface called irregularities or asperities. A load on the rubber slider causes the asperities to penetrate the rubber and the rubber drapes over the asperities. The energy needed to move the asperities in the rubber comes from the differential pressure across the asperities as shown in Fig. 3.4, where a rubber slider moves on an irregular surface at speed V.



Tearing and Wear

As deformation forces and sliding speeds go up, **local stress can exceed the tensile strength of the rubber**, especially at an increase in local stress near the point of a **sharp irregularity**. High local stress can **deform the internal structure of the rubber past the point of elastic recovery**. When polymer bonds and crosslinks are **stressed to failure the material can't recover completely**, and this can cause **tearing**. **Tearing absorbs energy**, resulting in **additional friction forces** in the contact surface.

Wear is the ultimate result of tearing.

$$F_{\text{total}} = F_{\text{adhesive}} + F_{\text{deformation}} + F_{\text{wear}}$$

Deformation Friction and Viscoelasticity

Rubber is elastic and conforms to surface irregularities. But rubber is **also viscoelastic**; it **doesn't rebound fully** after deformation.

Hysteresis

Hysteresis, or energy loss, in rubber.

where there is **some sliding** between the rubber and an irregular surface. If the **rubber recovers slowly** from the passing irregularity as in the high-hysteresis rubber, it **can't push on** the downstream surfaces of the irregularities **as hard** as it pushes on the upstream surfaces. This **pressure difference** between the **upstream and downstream faces of the irregularity** results in **friction forces** even when the surfaces are lubricated.

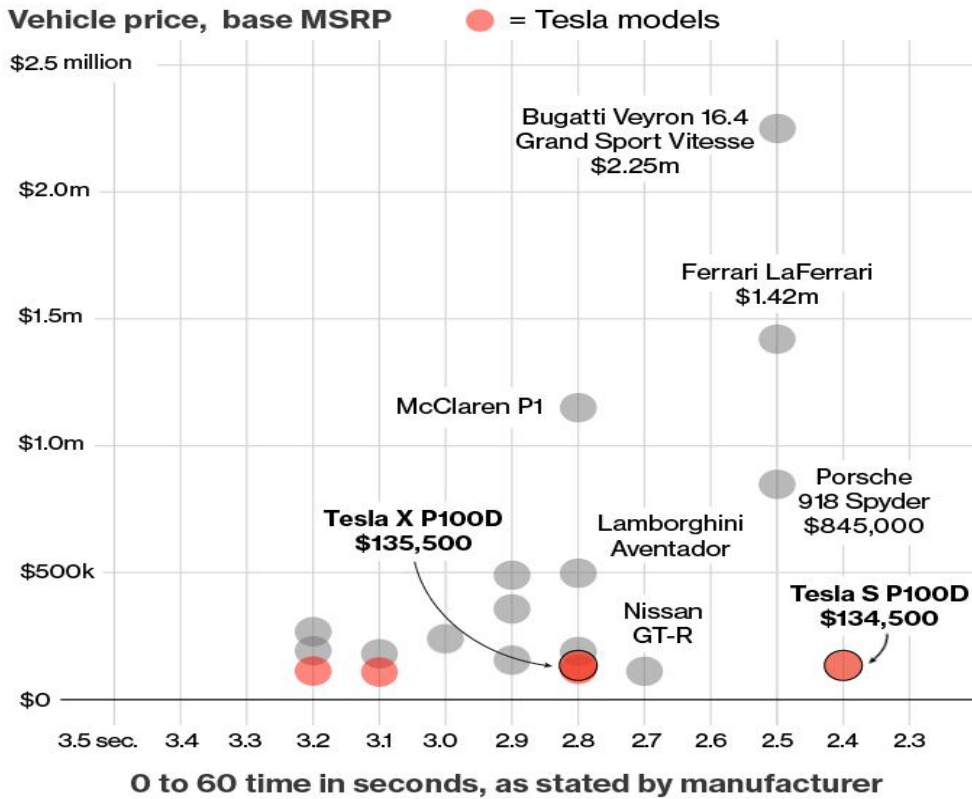
Wide Tires: It is true that wider tires commonly have better traction. The main reason why this is so does not relate to contact patch, however, but to **composition**. **Soft compound tires** are required to be **wider in order for the side-wall to support the weight** of the car. softer tires have a larger coefficient of friction, therefore better traction. A narrow, soft tire would not be strong enough, nor would it last very long. **Wear in a tire is related to contact patch**. **Harder compound tires wear much longer**, and can be narrower. They do, however have a lower coefficient of friction, therefore less traction. Among tires of the same type and composition, here is no appreciable difference in 'traction' with different widths. **Wider tires**, assuming all other factors are equal, commonly have **stiffer side-walls and experience less roll**. **This gives better cornering performance.**

Friction is proportional to the normal force of the asphalt acting upon the car tires. This force is simply equal to the weight which is distributed to each tire when the car is on level ground. Force can be stated as Pressure X Area. **For a wide tire, the area is large but the force per unit area is small** and vice versa. The force of friction is therefore the same whether the tire is wide or not. However, **asphalt is not a uniform surface**. Even with steamrollers to flatten the asphalt, the surface is still somewhat irregular, especially over the width of a tire. Drag racers can therefore **increase the probability or likelihood of making contact** with the road by using a wider tire. In addition a secondary benefit is that the wider tire increased the support base a

Friction force is independent of the apparent area of contact. For hard materials, this is nearly correct. The true area of contact varies with the applied load. The apparent area does not. If you can imagine the contact zone from a **microscopic viewpoint, only a tiny portion of the apparent area actually touches**. This tiny area is the true area of contact. But this applies to hard materials. It does **not apply to elastomers, such as rubber**. Tire tread rubber compounds vary greatly from one application to another. **Race car tire tread compounds can be very soft, viscoelastic materials**, while heavy truck tread rubber can be quite hard. In general, **soft rubber materials have greater friction**. With drag racing 'slicks', the tire tread material **literally sticks** to the pavement--and the rubber is **sheared from the tire**. Clearly, the greater the apparent contact area, the greater this shear force. **Cleanliness is important** to getting the surfaces to **'stick'**. This is one reason why drag racers have a **'burn-out'** before each race (another is to raise the tire tread surface temperature). However, there is another reason for wide tire treads on some road and track racing cars. They need **tread volume to provide enough wear life**. Tires wear rapidly under racing conditions. **Some long races wear out several sets of tires**. There are **trade-offs with traction and tread life**. That is why heavy truck tire tread compounds do not have as much friction as those used on passenger cars. However, truck tire tread compounds provide longer wear life and less heat build-up. Like many things in this world, **tire tread choices involve compromises.**

Tesla Versus the World's Fastest Supercars

Tesla's new P100D models accelerate as quickly as the fastest supercars from Bugatti, Ferrari and Porsche – at a fraction of the cost.



Sources: Vehicle manufacturers, Car and Driver reports

Bloomberg

